THE FORMATION OF A MYTH: GREEK MATHEMATICS—OUR MATHEMATICS

By JENS HØYRUP

Dedicated to
WALTER PURKERT
the friend

According to conventional wisdom, European mathematics (or, in the »strong« version, mathematics simpliciter) originated among the Greeks between the epochs of Thales and Euclid, was borrowed and well preserved by the Arabs in the early Middle Ages, and brought back to its authentic homeland by Europeans in the twelfth and thirteenth centuries (or, alternatively, it lay dormant in Byzantium and was brought to Italy by fleeing scholars at the fall of this city). Since then, it has pursued its career triumphantly.

This tale, more or less biased or false as an historical account, has none the less become material truth in the sense that it has contributed to the self-understanding and thereby to the cultural identity of the European mathematical community / communities for centuries. Tracing the emergence of the myth is thus in itself a contribution to the investigation of the formation of »European mathematics«. This (and not critique of the myth itself) will be the focus of the following.

I. SCHOLASTIC PRELUDE

An historian of mathematics easily gets the impression from the sources that neither Cicero nor Augustine but Isidore of Seville was »the second-most quoted authority next to the Bible during the Middle Ages«—and he may even suspect that the Bible come in second and Isidore first.

This is somehow paradoxical, since book III of Isidore's *Etymologies*, the one which covers mathematics, demonstrates that he knew nothing about the subject beyond some definitions and the gross heavenly movements. Yet the *Etymologies* conveyed the message that *mathematics is important*. In III,iv,1 we are told that

> Knowledge of numbers should not be held in contempt: In many passages of the Holy Scripture it elucidates the mysteries therein contained. Not in vain was it said in the praise of God: *You made everything in measure, in number, and in weight,*

and iv,3 goes on

> By number, we are not confounded but instructed. Take away number from everything, and everything perishes. Deprive the world of computation, and it will be seized by total blind ignorance, nor can be distinguished from the other animals the one who does not know how to calculate.[1]

That message was amply quoted by mathematically interested scholars during the High and Late Middle Ages.

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[1] PL 82, col. 155^b-156^b. Here as in the following, translations are mine when nothing else is stated.

Scholastic philosophers of this later epoch had other sources for their understanding of mathematics (beyond the actual mathematical works at their disposal). One was Aristotle, his use of mathematical illustrations of philosophical points no less than his direct discussion of the mathematical sciences. Other sources were found in encyclopedic works from the Islamic world, first of all in al-Fārābī's *Catalogue of the Sciences*, translated twice in the twelfth century by Gundisalvo and Gerard of Cremona and eclectically incorporated into Gundisalvo's own syncretic *De divisione philosophiae*. Manuscripts of these works were fairly wide-spread.

There is a recognized tendency in scholastic philosophy (though at the epoch largely subliminal) to focus on that part of the Islamic heritage which pointed back to classical Antiquity, either by developing the arguments of the Ancients or by real or fake ascriptions[2]. Among the opponents of the new learning of the twelfth and thirteenth century there was also an outspoken tendency to see it as »pagan learning«—but this category encompassed the Ancient pagans no less (much more explicitly indeed) than the Muslims. The champions of the new philosophy, though rightly or wrongly convinced that they were working within a tradition going back to the Ancients, would be quite willing to accept whatever learning of relevance for their pursuits they could get from the Islamic world as not only important but also true and legitimate. This can be illustrated by two passages, one from the twelfth and one from the thirteenth century. In 1159, John of Salisbury explained during a discussion of Aristotle's *Posterior Analytics*, that »demonstration« had

> practically fallen into disuse. At present demonstration is employed by practically no one except mathematicians, and even among the latter has come to be almost exclusively reserved to geometricians. The study of geometry is, however, not well-known among us, although this science is perhaps in greater use in the region of Iberia and the confines of Africa. For the peoples of Iberia and Africa employ geometry more than do any others; they use it as a tool in astronomy. The like is true of the Egyptians, as well as some of the peoples of Arabia.[3]

In the mid-1260es, Roger Bacon would speak in the *Opus majus* about

> the proposer of the law of wickedness, who is to come, as Albumazar teaches ..., and who in reality will be Antichrist; in order that all sects of pagans, idolaters, Tartars, heretics, and other unbelievers ... may be destroyed[4]

thus using the astrological science of Abū Maᶜšar to predict what should happen to Abū Maᶜšar and his fellow unbelievers. In the same vein, Roger suspected Byzantines, Muslims and Jews of falsifying the true knowledge contained in their books when

[2] This theme is discussed with references and examples in Høyrup 1988: 317-321.

[3] *Metalogicon* IV, vi (trans. McGarry 1971: 212).

[4] Trans. Burke 1928: 208, checked against the Latin text in Bridges 1897: 188.

»helping« ignorant translators[5].

Even when producing their Christian interpretation of Aristotle, Thomas Aquinas and Albertus Magnus would be strongly influenced by Avicennist and Averroist commentaries—and precisely ibn Sīnā and ibn Rušd »who made the great commentary« were lodged by Dante (*Inferno* IV, 106-144, ed. Blasucci 1965: 401) in the noble castle of Limbo along with the great philosophers, poets and rulers from Classical Antiquity, an honour which the Florentine poet would bestow on no other Muslim except Saladin the noble foe of the crusaders.

If philosophy in general was understood by the great schoolmen as a universalist category, this was even more true of mathematics[6] . Its roots were traced to the Greeks—no wonder, since even Islamic encyclopedias would treat the non-Greek pre-Islamic sources for mathematics anonymously. But no culture had an exclusive privilege of the Greek inheritance.

»Great schoolmen«, of course, were not the only scholars to have an opinion about mathematics[7], and the Isidorean idea that mathematics, *qua* liberal arts, were pivotal components in any scholarly culture deserving the name was not the only reason that mathematics was taken up by High Medieval scholars. Just as important was the role of mathematics *as a tool*,—not so much in mensuration, accounting and commerce as for understanding Nature. Medicine, astrology, and magic—perceived as insight into the secrets of Nature—were driving forces behind the explosion of what has come to be regarded as »scientific interest« from the late eleventh century onwards, but is perhaps better understood as »naturalism«.

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[5] *Compendium studii philosophiae* viii, ed. Brewer 1859: 472.

[6] Universalism, it must be said quite emphatically, did not entail any profound knowledge of the philosophy or mathematics of the Islamic world. Naïve belief that »the others« are »in reality not fundamentally different from us« was a hidden premiss, as it has been the hidden premise of much tolerance and universalism in later times.

That ingredient in the Greco-European myth which claims that Greek mathematics »was borrowed and well preserved by the Arabs in the early Middle Ages« can, indeed, be traced back to the High Medieval ignorance of the advanced level of Islamic mathematics. Al-Khwārizmī was translated and diffused; Abū Kāmil was translated into Latin, but left no trace beyond a few veiled references; al-Karajī was not translated, and what Leonardo Fibonacci borrowed from him for his *Liber abaci* was not adopted into the abacus school tradition; and not the faintest rumour about al-Samawʾal's sophisticated algebraic investigations appears to have reached the Latin world. In this way, the selective reception process took care that an image of Islamic learning as »in reality not fundamentally different« could emerge.

But much the same, it must be said with similar emphasis, happened to Greek philosophy and mathematics. The universalist perspective, though largely built on levelling illusions, remains universalist.

[7] For the moment I disregard the incipient tradition of non-scholarly practical »abacus school« computation—it will only become vaguely relevant for our purpose in the outgoing fifteenth century.

In these fields, the independent role of Islamic authors could hardly be overlooked. Retrospectively we may understand naturalism as an aspiration to gain insights into the working of the world (including in particular Life and Death and Future) which was independent of traditional Christian explanation[8], for which reason the pagan books may also have appeared particularly promising (and, it goes by itself, also dangerous as indeed the whole Faustian subject[9]). With no religious or counter-religious connotations, the technique of algorism (computation with »Arabic« numerals) was largely spread in the scholarly environment because of its value for astronomico-astrological computation[10]. That »algorism« was a discipline invented by a Muslim (Algorismus, i.e., al-Khwārizmī) was well-known, and that the numerals themselves came from India was told in the translated text.

The great mathematical translators of the twelfth century (Adelard of Bath, Gerard of Cremona, John of Spain, Plato of Tivoli, Hermann of Dalmatia) were motivated, partly by this naturalistic mood, partly by a general intellectual climate which tended to make them omnivorous. Thus Gerard, originally spurred by general dissatisfaction with the

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[8] In the fifteenth century, this feeling was still alive and given emphatic expression by Regiomontanus in the first of a series of lectures held in Padua on al-Farghānī. In the end of this lecture on the mathematical sciences and their utility, to which we shall return, Regiomontanus addresses the Numen of astrology in what can only be characterized as an anthem in prose, declaring it first to be »without doubt the most faithful herald of the immortal God who, interpreting his secrets, displays the Law according to which the Almighty resolved that the Heavens be made, on which he sprinkled the starry fires, testimonials of the Future«. Raising if not the tone (which was hardly possible) then at least the claim Regiomontanus goes on to tell that »this angelical doctrine makes us no less kindred of God than we are separated from the beasts by the other arts« (ed. Schmeidler 1972: 52). The latter phrase, we may observe, contains an echo of Isidore as quoted above.

[9] That astrology was considered dangerous in many quarters is quite clear, e.g., from Albertus Magnus's preface to his *Speculum astronomiae* (ed. Zambelli 1977; here pp. 5ff). There he complains that a number of books dealing with necromancy and other matters which are not »the root of sciences and inimical to true wisdom« parade as astronomy, making thus this whole subject suspect in the eyes of good people. Characteristically, however, in the ensuing attempt to distinguish good authors from frauds the former turn out to include, not only Ptolemy but also al-Zarqalī, al-Battānī, Thābit ibn Qurra, Jābir ibn Aflah, al-Bitrūjī, al-Farghānī, Abū Maᶜšar and many other scholars from the Islamic world. Hermetic and pseudo-Hermetic books, on the other hand, dominate the list of abominable books, while a book supposedly written by Aristotle to Alexander the Great is »the worst of all«.

[10] See, e.g., Petrus de Dacia's preface to his commentary on Sacrobosco's algorism (ed. F. S. Pedersen 1983: 81f). According to Sacrobosco, Petrus tells, the purpose of algorism is »better knowledge of everything«—»but I think that its more immediate end is astronomy: the practice of the present art is, indeed, an instrument for the investigation of the magnitudes of the heavenly movements«. It is characteristic that many *algorism*-treatises contain a second part dealing with (sexagesimal) fractions, only of use in the context of astronomy and astrology and indeed labelled »physical« [i.e., astronomical] or »philosophical fractions«.

limits of traditional Latin learning as well as by a specific desire to get hold of the *Almagest*, once he had arrived in Toledo stayed there for the rest of his life translating the treasures of the Arabs, according to an apparently well-informed fourteenth century biographical notice (ed. Boncompagni 1851: 387f). The only trace of a deviation from a fully universalist understanding of mathematics in the lists of their translations is a tendency on the part of John of Spain and Plato to translate works of Islamic and Jewish origin rather than translations from the Greek into Arabic (cf. Steinschneider 1904: 40-50, 62-66).

One of the two major mathematicians who were active in thirteenth century Europe—*viz* Leonardo Fibonacci—shares John's and Plato's predilection for material of non-Greek extraction and the tendency to work within the specifically Islamic domains (practical arithmetic, algebra, practical geometry) rather than the known Greek tradition. The other—Jordanus de Nemore—presents us with the exception from universalism which will turn out to confirm the rule while at the same time foreshadowing what was going to happen during the Renaissance[11].

Nothing is known about Jordanus's biography, but strong arguments can be given that he taught in Paris for a number of years somewhere between 1215 and 1245; here he will have been the centre of a circle of followers or associates, among whom Gerard of Brussels, Richard de Fournival, Campanus of Novara and in some way even Roger Bacon must be counted.

What is interesting about Jordanus for our purpose are his attitudes to mathematics and the aim of his enterprise as reflected in his works—only now and then directly told but regularly revealed by the relation between Jordanian and other works treating the same mathematical matters.

Jordanus recognized the mathematical superiority of the imports from the Islamic world. One of his works, the *Elementa arithmetica* in 10 books, rewrites the central subject of the traditional Latin quadrivium, (Boethian) arithmetic, so as to make it meet the metamathematical standards required by the imported learning, in particular the *Elements*. For this purpose, Jordanus invents a letter formalism allowing him to formulate proofs of the same generality as those of Euclid. A kind of sequel, *De numeris datis*, furnishes arithmetic with her own *Data*, making her thereby a perfect peer of Dame Geometry, whom the Muslims (as once the Greeks) had generally held in higher esteem. But there is more to it. The theorems of Jordanus's *Data* deal with what can be seen as the solvability of algebraic problems. »If a given number is separated into two parts the sum of whose squares is known, then each of the parts can be found« (I.4, trans. Hughes 1981: 128); »If the sum is known of the square of a number and the product of a given number and the root of the square, then the number can be found«

[11] The following presentation of Jordanus and his role and ideas draws on Høyrup 1988, where detailed documentation can be found.

(IV.8, *ibid.* p. 168). To some extent this might be a calque from Euclid's geometrical *Data* (thus I.4). But a number of theorems are so characteristic that their similarity to problems known from Islamic algebra's cannot be accidental. IV.8 will already be recognized as the first mixed case from al-Khwārizmī's *Algebra*[12]. Other theorems appear directly grotesque, like I.25, which (in modern symbols) tell that x and y can be found from $x+y=A$, $^{1}/_{N}\cdot{}^{Cx}/_{y}+Cx=D$ if A, C, D and N are given, and which coincides with another problem from al-Khwārizmī's work (»various questions«, N° 5; Hughes 1986: 252). The presence of both types in *De numeris datis* implies that Jordanus's aim was, firstly, to construct a metatheoretically satisfactory theory (an *apodixis*, to follow the Aristotelian terminology used by Richard de Fournival to characterize Jordanus's works[13]) encompassing and explaining the results of Islamic algebra (which, in Richard's terminology, consisted of mere *experimenta*); secondly, to demonstrate that his theory was really able to do *anything* which could be accomplished by means of algebra. In order to make quite sure that this message was understood Jordanus provided the theorems with numerical examples, which often (e.g., in I.4 and I.25) coincide conspicuously with those used by the Islamic precursors (beyond al-Khwārizmī also Abū Kāmil either directly or through Fibonacci, possibly also al-Karajī).

In the second part of his *Demonstratio de algorismus* (both the original, short, and the longer, expanded version[14]), where sexagesimal fractions are generalized into what Jordanus calls »consimilar fractions« $\Sigma a_n \cdot p^{-n}$ (sexagesimal for p=60, decimal for p=10), Jordanus generalizes further to »dissimilar fractions« of the type

$$\frac{a_1}{P} + \frac{a_2}{P\cdot Q} + \frac{a_3}{P\cdot Q\cdot R} + \ldots \quad .$$

The introduction of the dissimilar fractions amounts to a genuine naturalization of the »ascending continued fractions« which were used amply in Islamic mathematics. This root of the concept, however, is completely camouflaged: as sole justification, Jordanus offers an extensive array of examples referring to Latin metrology as known from Isidore (who is mentioned explicitly), Hrabanus Maurus and other traditional sources. None of these, it should be observed, had ever discussed metrological sequences in terms of composite fractions[15]. Once again, Jordanus appreciates the superiority of the mathematical knowledge of the Muslims and silently acknowledges its utility while denying its legitimacy. What results from his act of naturalization is an implicit falsification of history which makes (his brand of) Latin-European mathematics a direct

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[12] Accessible to Jordanus, we have reasons to believe, in Gerard of Cremona's translation (ed. Hughes 1986).

[13] *Biblionomia* N° 45, ed. Delisle 1874: 526.

[14] The essential excerpts of both are found in Eneström 1913a.

[15] The idea is indeed so strange to the tradition to which Jordanus refers it that Eneström misunderstands the text which he publishes.

continuation of Ancient mathematics. This is in full agreement with the principle enunciated in the preface to the first algorism on integers, where, after a set of definitions in Euclidean style, we are told to »continue in the footsteps of the Ancients« (ed. Eneström 1907: 139), while the Arabs, the Indians, and al-Khwārizmī himself go unmentioned.

A final work where Jordanus distinguishes himself clearly from the ways of Islamic authors (while *possibly* borrowing even in this case) is in his *De plana sphaera* on the theory of the astrolabe. Ancient spherics (Autolycos, Menelaos, Theodosios) had been abstract in form, and never told directly that the sphere and great circles with which it dealt were the heavenly sphere, the horizon, the equator, etc. Islamic commentators had mostly changed that, telling what the theory was *really* meant to be about[16]; Ptolemy's *Planisphaerium* told so too, at least the Latin translation from the Arabic which is the only extant version of the work. Contrary to this background, Jordanus's work on the subject is totally free of all explicit references to astronomical categories.

Jordanus is thus an exception to the rule that High Medieval learning had a universalist attitude to mathematics. That he is in fact a confirmatory exception follows from what happened to the Jordanian inspiration.

The first thing to happen to *De plana sphaera* was that new, expanded versions were prepared which agreed with the current style[17]; one thirteenth century Master Gernardus wrote an *Algorismus demonstratus* (ed. Eneström 1912, 1913) which took over Jordanus's letter formalism and the rigour of his proofs but dropped the generalization of sexagesimal into consimilar fractions as well as the dissimilar / ascending continued fractions. Algebra, finally, when taken up by the fourteenth century university scholar Jean de Murs in his *Quadripartitum numerorum*, was borrowed from Leonardo's *Liber abaci* (see L'Huillier 1980), while Jordanus's *De numeris datis* seems to have had no influence whatsoever. Richard de Fournival, who appears to have been intimately familiar with Jordanus's works and close to his circle, was as universalist and naturalist as anybody, according to the impressive list of works he bought and had copied for his library; though his descriptions of Jordanus's works show that he must have understood Jordanus's specific ideals, he did not share them.

Curiously enough, the Medieval scholastic university *did* produce an unprecedented, and hence specifically European kind of mathematics, in connection with the »quantification of qualities« and the whole mathematization of philosophy, from Bradwardine to Oresme. This was seen, however, as part of the general *via moderna* in philosophy. Thus, in 1346, Nicholas de Autrecourt was condemned by the Roman Curia, among many other things for maintaining that the little which could be known

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[16] Cf. Matvievskaya 1981.

[17] See the parallel editions in Thomson 1978.

for sure about natural matters could be understood quickly if people would direct their intellect to the things themselves and not to the doctrines of Aristotle and ibn Rušd (ed. Denifle & Chatelain 1891: 580); the ensuing general exhortation reproaches those Ockamists who pursued the barren modern road in philosophy in contempt of Aristotle and other old masters and expositions (ibid., p. 588). The formulations strongly suggest a distinction between *modern Latin* and *traditional Greco-Islamic* thought. Oresme's twin works *Algorismus proportionum* and *De proportionibus proportionum*[18] also show that the new mathematical structure which he investigates stands with one foot on the shoulder of Islamic algorism and the other on that of Euclidean theoretical arithmetic.

This highly sophisticated and autochthonously Latin-European kind of mathematics was thus not understood by its contemporaries as belonging within a specifically Greco-European current of thought. It was rightly seen as setting fourteenth century thinking equally apart from all precursors.

II. HUMANISM AND CIVIC MATHEMATICS

The idea that (some kind of) Europe had special inheritance rights with respect to Ancient thinking originated with the Italian Humanists. The progenitors of the notion that this concerned even the relation between »European« and Greek mathematics knew little of either, in particular when compared to near-contemporary minds like Oresme.

Petrarch, it is true, did not directly include mathematics when taking privileged possession of Ancient thought and letters. None the less, he is of importance. Parodically we may say that he did not know a single of the works by Archimenides (and what other forms of the name the double translation Greek-Arab, Arab-Latin had produced) that had been studied by thirteenth century scholars; but Petrarch, in contrast to many university scholars of the thirteenth century, knew how to spell the name of their author. More in earnest: By writing several short biographies of Archimedes (printed in Clagett 1978: 1336-1339), Petrarch transmitted the Ancient awe for this figure to Humanist culture; veneration for the man invited interest in his work—and gradually, as it was discovered that Archimedes was not only a wondrous engineer with a high civic spirit but the producer of actual mathematical theorems and techniques, Humanist »Archimedism« furthered the acceptance of mathematics as a legitimate and even vital part of (Ancient, and therefore real/Humanist) culture[19]. Evidently, far from all Humanists went so far; but some did, and mathematically engaged participants in the Humanist movement had a weighty argument at their disposal.

[18] See Høyrup 1987: 35-37.

[19] This process is dealt with in Høyrup 1992.

One of these was Alberti. His work on perspective shows him to possess sound geometrical understanding; but he was not a man of broad mathematical culture, and (given his interests) astonishingly ignorant of what others had accomplished in mathematics during the Middle Ages. His *Ludi matematici* owe much to the mixed tradition of practical geometry, and nothing to what he may have known about Ancient mathematics. Yet in the dedicatory letter of the Italian version of the *De pictura* he tells the mathematical sciences to be among those »elevated and divine arts and sciences« that had flourished in Antiquity but were now »missing and almost completely lost«: painting, sculpture, architecture, music, geometry, rhetorics, augury and »similar noble and wonderful« undertakings (ed. Grayson 1973: 7).

Slightly later in date is Regiomontanus, whose original main interest was astronomical, but whose intercourse with Bessarion's Humanist circle in Rome oriented him broadly toward Ancient mathematics. He was much too great a mathematician to claim that nothing of value had been produced in mathematics since Antiquity; his publishing plans (interrupted by his sudden death) included, beyond the Ancients (Ptolemy, Euclid, Theon, Proclos, Firmicus Maternus, Archimedes, Menelaos, Theodosios, Apollonios, Hero and Hyginus), several works from the Latin thirteenth and fourteenth century: Jordanus's *Elementa arithmetica* and *De numeris datis*, Witelo's *Perspectiva*, and Jean de Murs' *Quadripartitum numerorum*[20]. More detailed information about his views can be extracted from his introductory Padua lecture from 1463/64 (ed. Schmeidler 1972: 43-53). Mathematics proper consists of two branches: the study of quantity—geometry; and the study of number—arithmetic. The former arose when people began asking general questions inspired by Egyptian surveying. Euclid (identified with Euclid of Megara) collected the scattered material and added much of his own in the 13 books of the *Elements*, which were transmitted to the Latin world by Boethius, Adelard, King Alfred, and Campanus; even though the distinct characters of the different translations are described, no word is wasted on the Arabic background to Adelard's translations. Further on, Archimedes and Apollonios are discussed at length and Eutocios, Theodosios and Menelaos mentioned briefly. Other brilliant authors writing on geometry »in various languages« are bypassed in silence »for lack of time«.

Concerning arithmetic we are told that even though Pythagoras left important knowledge in part borrowed from itinerant Egyptian and Arab (*sic*) teachers, Euclid gave the subject a much more valuable treatment in *Elements* VII-IX, from which Jordanus picked his 10 books on arithmetic, adding his most beautiful *De numeris datis*. On the »flower of arithmetic«, »the art of *res* and *census*, which today is called by the Arabic name algebra«, Diophantos wrote 13 books. To this comes the *Quadripartitum numerorum* and Nicomachos's *Arithmetic* translated by Boethius. Algorism is represented by *Algorismus demonstratus* and by the thirteenth century Greco-Italian Barlaam, while

[20] This according to an advertizing circular reprinted in Schmeidler 1972: 532.

nothing is said about the role of Indians and Arabs.

This silence has little to do with general dismissal of non-»European« contributions to the sciences. When it comes to the »intermediate sciences« (astronomy, music, perspective, and less common subjects like the science of weights, aqueduct construction, and the proportion of velocities in movement—a reference to the Bradwardine tradition) Regiomontanus is quite willing to present Arabic authors (and Indians and Persians anonymously) when they are known to him and pertinent. This is particularly the case in the discussion of astronomy, which Regiomontanus personally ranks much higher than mathematics proper, next only to astrology; he even quotes Albertus Magnus with approval for the characterization of Jābir ibn Aflaḥ as the »rectifier of Ptolemy«.

There is thus a tendency in Regiomontanus's text to consider precisely *mathematics proper* as a field only cultivated adequately by Greeks and Latins. There is also a tendency, through reference to specific nations (Britons, Gauls, Germans though only obliquely, Italians, Hungarians), to approach a modern concept of Europe.

This picture of mathematics disagrees too sharply with Regiomontanus's actual knowledge to have been empirically derived. Firstly, before recognizing *algebra* in Diophantos he must have known not only the name but the substance of Arabic algebra. Secondly, the ascription of *algorism* to an Arabic author of that name was as familiar as the idea that the numerals themselves came from the Indians. Thirdly, Regiomontanus knew perfectly well to esteem Jābir not only as an astronomer and a rectifier of Ptolemy but also as a contributor to the field of spherics (which he had counted as geometry, cf. the references to Menelaos and Theodosios), since he turns out to have borrowed quite freely from him for his own *De triangulis* (cf. Lorch, "Jābir ibn Aflaḥ", *DSB* VII, 39). Seeing mathematics proper as a purely Greco-Latin business but astronomy as Greco-Arabo-Latin must have involved an appreciable amount of preconceived ideas and double-think on the part of the first really significant mathematician affiliated with the Humanist current.

Regiomontanus's lecture is told to deal with »the mathematical sciences, and their utility«, and even though Regiomontanus only mentions the utility in the mechanical arts, in commerce and in war in the most general terms before taking up the importance of mathematics for liberal studies, it still seems paradoxical that the characteristic contributions of the Islamic tradition—the integration of theory and applications—are dismissed so cavalierly. *Utility*, however, is a key-word among Humanist mathematicians in general; closer reading of the texts which specify the concept allow us to decipher it as *civic utility*, which endows utility for courtly preoccupations like visual arts and astrology, in warfare and for philosophy with higher letters of nobility than service for *roturier* trade and surveying—on their part more highly regarded than the application of mathematics to non-military mechanical arts (a hierarchical ladder which Regiomontanus follows in his text).

From the beginning, Humanism was a civic movement, tightly connected with urban patrician and (when it arose) courtly culture. Similarly from the beginning (i.e., from Petrarch), *civic utility* was the most outstanding reason for its interest in Archimedes. In spite of Plutarch's much-read effort to convince his readers that only Hieron's pushing persuasion made Archimedes divert a little of his theoretical genius to military arts, and that Archimedes was of too high spirit to leave anything written on so base matters (*Vita Marcelli*, ed. Carena 1981: II, 331, 335), »Archimedism«, beyond its other functions, served to screen the importance of precisely the Islamic tradition for that kind of mathematics which the Humanists would praise.

The Italian »abacus school« had mainly taught such uses of mathematics which were located at the bottom of the civic hierarchy: commercial arithmetic, surveying and similar practical geometry. According to those *Libri d'abbaco* which I have investigated it had been preciously little concerned with speculations about the origin and the moral/ideological evaluation of mathematics, and no more with Archimedes nor any other mathematician of renown. At the moment when mathematicians with a background in the abacus tradition wanted to justify their subject *vis-à-vis* Humanist or courtly circles[21], however, Archimedes was heavily appealed to—thus in Luca Pacioli's *De divina proportione* (ed. Winterberg 1896)—which almost by necessity produced the Greco-Latin picture of mathematic.

But only *almost*. That fervent reverence for Archimedes could be compatible with a more open mind even in the sixteenth century was demonstrated time and again by Cardano. In the *Encomium geometriae* (Cardano 1663a: 440-445), read in the Academia Platina in Milan in 1535, he not only tells about the Phoenician and Egyptian roots of arithmetic and geometry (evident to anybody who had read Proclos's just-published introduction to *Elements* I). He also tells about al-Kindī's »concise and most beautiful« work on proportions[22], about al-Khwārizmī's »mixed discipline called algebra«, and about Jābir ibn Aflah's treatise on circles and squares. Just as unusual is his reference to the authors of the two most impressive works from the abacus school tradition, Leonardo Fibonacci's *Liber abaci* and Luca Pacioli's *Summa de arithmetica*.

In book XVI of his *De subtilitate*, a similar list is found (Cardano 1663: 607f). Even here, al-Khwārizmī, al-Kindī and Jābir ibn Aflaḥ occur in the company of Archimedes, Euclid, Swineshead, Apollonios, and Archytas, accompanied by a few non-mathematicians. In the same pages Cardano is seen to refer to a Mediterranean, not to a European *we:* after speaking about John Scot and Swineshead he remarks that after this he »believes that there is hardly any reason to doubt what is written in the book *On the*

[21] Largely the same in this respect. As discussed by Biagioli with particular focus on the Urbino mathematicians, justifying the legitimacy of their subject amounted to much the same as elevating the social position of the mathematical professions.

[22] Al-Kindī's work on proportions and composite medicines is perhaps, we may note in passing, the closest one can come to an direct inspiration for the fourteenth century mathematization of philosophy—see McVaugh 1967, and Sylla 1971: 17-23.

Immortality of the Soul, namely that the barbarians are not detectably our inferiors in intellect«, given that the foggy British heaven gave rise to two such brilliant minds.

On two accounts, an open and informed mind like Cardano's was thus unable to perceive a specific (Greco-)European mathematics in 1550. Firstly: Outside Italy and the German area (including Gemma Frisius's Netherlands), mathematics could with some right be considered a subject dead since almost 200 years[23]. Only Recorde's *Whetstone of Witte* (1557), Nunes *Livro de Algebra* (1567), Dee's and Foix de Candale's work on the *Elements* including the *Praeface* of the former (1570) and the stereometric supplementary books of the latter (1566/1578) were to inaugurate an era where mathematics was no longer remarkably *less* common-European than Humanist culture or scholarly culture in general. Secondly: Several fields, most outstandingly algebra but also optics[24] and astronomy, were still so close to well-known Islamic ancestors—both in their style and in what they actually knew and were able to accomplish—that only the dishonest or the ignorant could overlook this descent.

III. DISHONEST IGNORANCE BECOMING CONVENIENT GULLIBILITY

Dishonest and ignorant, Petrus Ramus was precisely enough of both, and enough of a preacher to make ignorance and dishonesty serve his purpose. *He* was thus able to formulate the myth of Greco-European mathematics conclusively at a moment where it was still in visible disagreement with contemporary facts.

In 1569 he published his *Scholae mathematicae*. The first three books are dedicated to Catharina of Medici and meant to persuade her to further the mathematical professions in a way which everybody else would have found outrageously lavish until the Sputnik shock. Already the first page of the dedicatory letter refers twice to Europe as a coherent totality; in the end it tells that France, truly, is going to be the first but Europe as a whole the ultimate beneficiary of the grandiose programme.

So far *European mathematics* is thus something still to be created. Book I, however, on the history of mathematics since Adam, shows that nobody but the Patriarch's, the

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[23] The only noteworthy exceptions had been dead-ends. The Provençal-French algebraic school culminating in Chuquet's *Triparty* (see van Egmond 1988) had vanished without leaving noticeable traces, and the high point of contemporary French mathematics was Oronce Fine—best characterized by the fact that he had the section on practical geometry from Gregor Reisch's *Margarita philosophica* republished in 1549. Portuguese navigational mathematics had been too much of a military secret before Pedro Nunes to be widely diffused.

[24] As late as 1572, Risner made a common edition of ibn al-Haytham's and Witelo's optics, clearly not from antiquarian interest but because these were deemed fundamental works.

Greeks, and (appearing in books II-III) modern Europeans ever made anything worth mentioning in mathematics—with the one exception that Abraham once taught the Egyptians about the subject[25], which helps Ramus explain away the references of Greek authors to the Egyptian origin of geometry and the Phoenician invention of arithmetic.

Apart from a Tom-Lehrer-like joke quoted in the dedicatory letter, »Maybe the Turks won all the battles, but we stole all the good books« [of the Byzantines], the Islamic world only appears in a reference (p. 37) to the »art of complex arithmetical subtlety which popularly is called by the Arabic name algebra«. In the same breath this implicit ascription is declared to be false, since we owe the art in question to Diophantos.

This argument had already been used a century earlier by Regiomontanus, we remember. Regiomontanus, however, knew something about Diophantos's *Arithmetic* from his correspondent Bianchini and was able to appreciate its algebraic character. Ramus too knew about algebra, and wrote a book about it (1560). Apart from Ramus's habitual introduction of graphic schemes, however, this book is fully dependent on the Islamic tradition, and hardly a single real step in advance of al-Khwārizmī and his twelfth century translators[26]. Algebra is made »ancient« through the identification of the sequence of powers with a continued proportion (familiar from the *Elements*) and by making the introductory first-degree problem (fol. 11^{v}) deal with the age of Alexander the Great and three of his followers[27].

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[25] The Wisdom of the Patriarchs is not Ramus's own invention. He had found in Josephus's *Archaeologica* a source which fitted his purpose. In general, if only they are adequate he uses sources freely which contemporaries would deem irrelevant.

[26] Knowledge of Stifel's *Arithmetica integra*, in particular the chapter "De Algorithmo numerorum Cossicorum" (Stifel 1544: fol. 234^{v}-239^{v}) may be the reason that Ramus lists names for powers (exemplified by 2) until 2^{20} (without going beyond the second degree in actual algebra). The same chapter, or possibly some intermediate work, appears to be the source for his graphical schemes. The rule for solving mixed normalized second-degree equations is justified with a reference to *Elements* II.4 (similarly to what is done by Abū Kāmil, Savasorda and Fibonacci, who however use II.5)—but the choice of words (... 1q[uadratus]+8l[ati] aequatur gnomoni ...) show that al-Khwārizmī's diagram (repeated by almost every algebrist until Cardano and Stifel) and not Euclid is on Ramus's mind.

[27] Once again, Stifel's *Arithmetica integra* may be the immediate source for both ideas. Even he identifies his »cossic progression« as »geometric« (fol. 234^{v}), which is of course near at hand. More striking is the appearance (fol 234r) of the problem of ages, sharing not only the numbers and the four characteristic names but also so much of the phrasing that direct copying seems plausible.

The second edition of Ramus's *Algebra*, published by Lazarus Schoner in Frankfurt in 1586, has a rather different approach to the origin of algebra (quoted by Giovanna Cifoletti in the present volume), involving »the erudite nations of the East«, Syrians and Indians together with Alexander the Great, phrased in a Latin style which differs strongly from Ramus's. A number of similarities with a somewhat earlier German *Algebra des Initius Algebras* (ed. Curtze 1902: II, 449-609; earliest manuscript 1545), including the ascription of the terms *Aliabra und Aluoreth* to

Ramus's ignorance is dishonest in the sense that he was wrong; that he could have known that he was wrong—should have known in view of his broad culture; and that he may secretly have known that he was wrong. Well-informed contemporaries, in fact, knew better—one year after the appearance of the *Scholae mathematicae*, Dee and Commandino published a translation of a work *On the division of surfaces* »Machometo Bagdadino ascriptus«, which they presumed to be a version of Euclid's treatise on the same subject; in 1572, as mentioned above, ibn al-Haytham's *Optics* was printed; and in 1594 an Arabic version of the *Elements* (supposed to be) by Naṣīr al-Dīn al-Ṭūsī appeared in Rome.

Most members of Ramus's audience were of course no better informed than their master intended to be. Within a generation, furthermore, »European-wide mathematics« had become an actual reality producing quite new insights and formalisms, which made what had once been a strained intellectual construction look credible. *Algebra*, the key stumbling stone of the myth, was so radically transformed by Viète under *really Greek* inspiration that nobody can blame him much for speaking about his work as »a new art, or rather so old and so defiled and polluted by barbarians that I have found it necessary to bring it into, and invent, a completely new form« (ed. Hofmann 1970: xi). Within another generation Descartes sparked off another revolution in the field so effective that not only al-Khwārizmī but also the abacus school and cossist algebra could be safely forgotten by mathematicians and advanced practical calculators alike, while Cardano's *Ars magna* had become incomprehensible unless its results were rewritten in the new symbolism.

The rise of »European mathematics« as actual reality did not in itself make the acceptance of the Greco-European myth compulsory. Seventeenth century European mathematics might well have found itself to deviate radically from the Ancient canons—one need only think of the victorious non-rigorous trend in the treatment of infinitesimals, and on the integration of mathematics with experimental philosophy. It might even, had it been really well-informed on the character of Islamic science (but only bold extrapolation from ibn al-Haytham's *Optics* would have allowed it to be) have discovered itself as a continuation and fulfillment of the promises of the Islamic ninth to twelfth centuries (CE). In the age of the incipient colonial expansion, however, such alternative histories or myths would have seemed awkward, perhaps even improper[28].

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the Indians, suggest that Schoner has drawn upon this or related German traditions for his admitted »emendations and explications«.

[28] In terms of the discussion of note 6 we may observe that the early sixteenth century was still able to discuss the attitudes to life of the inhabitants of America in terms of familiar philosophy—rather Epicureans than Stoics, as Amerigo Vespucci tells (quoted in Turner 1965: 136)—seeing them thus as »in reality not fundamentally different from us«. At that moment universalism similar to that of the scholastics toward Islamic philosophy and mathematics had still been possible. Fifty years later it was not.

Since not only the foreign but also the socially »low« ancestry of mathematics was disavowed

The myth so fittingly prepared by Humanist mathematicians for a different purpose, to the contrary, was conveniently at hand and was generally adopted and handed down until the present or almost-present time. So conveniently, indeed, that we may perhaps be allowed to twist a famous Libertine gibe, to the effect that »s'il n'existait pas, il aurait fallu l'inventer«.

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and veiled by the myth of practical mathematics being a mere adaptive application of scientific mathematics (even on this account Ramus's *Scholae* are characteristic, in spite of his vociferously outspoken advertising of utility and his contempt for pure theory), one may suggest that not only colonialist depreciation of the non-European but also »class-struggle from above« is obliquely reflected in the self-image of post-Renaissance mathematics (as obliquely reflected as class-struggle always is in the formation of attitudes, but none the less a reflection of the process in which the patriciates transformed themselves into *noblesse de robe* or upper bourgeoisie while reducing the artisanate into a section of the working class and depriving it of its cultural autonomy).

In this connection, a more general observation on a shift in European culture in the later sixteenth century deserves to be quoted. It was made by Carlo Ginzburg (1980: 126) and takes it starting point in »a problem the significance of which is only now beginning to be recognized: that of the popular roots of a considerable part of high European culture, both medieval and postmedieval. Such figures as Rabelais and Brueghel probably weren't unusual exceptions. At the same time, they closed an era characterized by hidden but fruitful exchanges, moving in both directions between high and popular cultures. The subsequent period was marked, instead, by an increasingly rigid distinction between the culture of the dominant classes and artisan and peasant cultures, as well as by the indoctrination of the masses from above. We can place the break between these two periods in the second half of the sixteenth century, basically coinciding with the intensification of social differentiation under the impulse of the price revolution. But the decisive crisis had occurred a few decades before, with the Peasants' War and the reign of the Anabaptists in Münster. At that time, while maintaining and even emphasizing the distance between the classes, the necessity of reconquering, ideologically as well as physically, the masses threatening to break loose from every sort of control from above was dramatically brought home to the dominant classes.

This renewed effort to achieve hegemony took various forms in different parts of Europe, but the evangelization of the countryside by the Jesuits and the capillary religious organization based on the family, achieved by the Protestant churches, can be traced to a single current. In terms of repression, the intensification of witchcraft trials and the rigid control of such marginal groups as vagabonds and gypsies corresponded to it«.

An unexpected cluster of siblings to the new-born *European mathematics* of the late Renaissance: Jesuit Counter-Reformation, Lutheran and Puritan orthodoxy and self-repression, witch- and gypsy-hunting. But unlike its brothers and sisters, *mathematics* continued to draw advantage from its lowly connections. In agreement with the classical definition of a South-State gentleman, European mathematics would gladly share the bed of attractive non-European and plebeian disciplines—but it would never stoop to having its academic breakfast in their company.

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